Uitwerking 18-3-'98

① ⓐ $\int_{-\infty}^{\infty} F(x)\,dx = \int_{0}^{\infty} A x e^{-\frac{x}{\tau}}\,dx = A\tau^2 = 1 \Rightarrow A = \frac{1}{\tau^2}$

ⓑ $\mu = \int_{-\infty}^{\infty} x F(x)\,dx = \int_{0}^{\infty} A x^2 e^{-\frac{x}{\tau}}\,dx = 2A\tau^3 = 2\tau$

ⓒ $\sigma^2 = \overline{x^2} - \overline{x}^2$

$\overline{x^2} = \int_{-\infty}^{\infty} x^2 F(x)\,dx = \int_{0}^{\infty} A x^3 e^{-\frac{x}{\tau}}\,dx = 6A\tau^4 = 6\tau^2$

$\overline{x}^2 = \mu^2 = 4\tau^2$

$\Rightarrow \sigma^2 = 2\tau^2 \Rightarrow \sigma = \sqrt{2}\,\tau = \frac{1}{\sqrt{2}}\,\mu \Rightarrow \sigma$ evenredig met gemiddelde

ⓓ $P\{0 \le x \le \tau\} = \int_{0}^{\tau} F(x)\,dx = \int_{0}^{\tau} A x e^{-\frac{x}{\tau}}\,dx = -A x \tau e^{-\frac{x}{\tau}}\Big|_0^{\tau} - \int_{0}^{\tau} A\left(-\tau e^{-\frac{x}{\tau}}\right)dx = -A\tau^2 e^{-1} + A\tau\left[-\tau e^{-\frac{x}{\tau}}\right]_0^{\tau} =$

$-A\tau^2 e^{-1} - A\tau^2 e^{-1} + A\tau^2 = -2e^{-1} + 1$

($A = \frac{1}{\tau^2}$)

② ⓐ $S_I^2 = \left(\frac{\partial I}{\partial M}\right)^2 S_M^2 + \left(\frac{\partial I}{\partial a}\right)^2 S_a^2 + \left(\frac{\partial I}{\partial b}\right)^2 S_b^2$

ⓑ $= \left(\frac{1}{12}[a^2+b^2]\right)^2 S_M^2 + \left[\frac{1}{6}Ma\right)^2 S_a^2 + \left(\frac{1}{6}Mb\right)^2 S_b^2$

$\Rightarrow S_I = 3{,}4 \cdot 10^3\ g\,cm^2$

$I = (28{,}3 \pm 3{,}4) \cdot 10^3\ g\,cm^2$

ⓒ $\overline{I} = \frac{I_P w_P + I_Q w_Q}{w_P + w_Q}$ met $w_i = \frac{1}{S_i^2}$

$\overline{I} = 28{,}6 \cdot 10^3\ g\,cm^2$

ⓓ $\frac{1}{S_{\overline{I}}^2} = \frac{1}{S_{I_P}^2} + \frac{1}{S_{I_Q}^2} \Rightarrow S_{\overline{I}} = 2{,}5 \cdot 10^3\ g\,cm^2$

③ ⓐ $I = \frac{V}{R} + C = aV + C$ met $a = \frac{1}{R}$

a $M = \sum w_i D_i^2 =$ minimaal $= \sum w_i (I_i - aV_i - C)^2$

$\frac{\partial}{\partial a}$: $a\sum w_i V_i^2 + C\sum w_i V_i - \sum w_i V_i I_i = 0$

$\frac{\partial}{\partial C}$: $a\sum w_i V_i + C\sum w_i - \sum w_i I_i = 0$

$\Rightarrow$

$a = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2}$ met $\bar{x} = \frac{\sum w_i x_i}{\sum w_i}$ en $w_i = \frac{1}{S_i^2}$

Uitwerking 18-3-1gB

③ (a) $q = \dfrac{\overline{VI} - \overline{V}\,\overline{I}}{\overline{V^2} - \overline{V}^2}$ met $\overline{V} = \dfrac{\sum w_i V_i}{\sum w_i}$ en $w_i = \dfrac{1}{s_i^2}$

$$\Rightarrow R = \frac{\overline{V^2} - \overline{V}^2}{\overline{VI} - \overline{V}\,\overline{I}}$$

(b) geen fout gegeven $\overline{V} = \dfrac{\sum V_i}{N}$

$\overline{V} = \frac{1}{4}(2{,}0 + 4{,}0 + 6{,}0 + 8{,}0) = 5{,}0$ V

$\overline{V}^2 = 25$ V

$\overline{V^2} = \frac{1}{4}(4{,}0 + 16 + 36 + 64) = 31$ V

$\overline{I} = \frac{1}{4}(1{,}5 + 2{,}0 + 2{,}6 + 3{,}0) = 2{,}3$ μA

$\overline{VI} = \frac{1}{4}(3{,}0 + 8{,}0 + 15{,}6 + 24) = 13$ V μA

$$R = \frac{31 - 25}{13 - 5{,}0 \cdot 2{,}3} = 4{,}0 \cdot \text{k}\Omega$$